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# D-brane effective actions and particle production near the beginning of tachyon condensation

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Abstract: In this paper we will study the quantum field theory of fluctuation modes around the classical solution that describes tachyon condensation on unstable D-brane. We will calculate the number of particle produced at times near the beginning of the rolling tachyon process. We will perform this calculation for different tachyon effective actions and we will find that the rate of particle production strongly depends on the form of the effective action used for the description of the early stage of the tachyon condensation.

Keywords: D-branes, Tachyon Condensation.

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## 1. Introduction

One of the most intriguing problems in string theory is understanding tachyon condensation and its description in terms of tachyon effective theory. Generally, it seems to be difficult task to find effective action for the tachyon field only since the scale of masses of an infinite set of massive string modes is the same as that of the tachyon mass. Consequently keeping the tachyon and integrating out all other string modes seems to be not well justified. On the other hand we can still believe that in certain situations (like non-BPS D-brane decay, forming of the tachyon kinks and vertex) some aspects of string dynamics can be described by an effective field theory action involving only tachyon field and massless modes where all other massive modes effectively decouples at the vicinity of certain conformal points.

In recent years there were many papers that discussed various forms of tachyon effective actions [1]–[16]. Explanation, why there are so many proposals for tachyon effective actions was given recently in very interesting paper [17]. As was stressed there the form of the tachyon effective action may depend on a choice of a region in field space where it should be valid. One such a region corresponds to the neighbourhood of the perturbative string vacuum $T = 0$ where we can reconstruct tachyon effective action from string $S$-matrix. Then we can expect the effective action

$$S = -\int dt d^p \mathbf{x} \left( \frac{1}{2}\eta^{\mu\nu}\partial_\mu T \partial_\nu T - \frac{\mu^2}{2} T^2 + g_1 T^4 \dots \right) . \tag{1.1}$$

Another possibility how to get tachyon effective action is to study tachyon dynamics near end point of tachyon evolution $T \gg 1$ where tachyon gets frozen. Then we can find from the derivative expansion of the superstring partition function on the disc [18]

$$S = -\int dtd^p\mathbf{x}e^{-\frac{1}{4}T^2}\left[1+\frac{1}{2}(1+b_1T^2)\eta^{\mu\nu}\partial_\mu T\partial_\nu T+\dots\right], \qquad b_1 = \ln 2 - \frac{1}{2}. \qquad (1.2)$$

One may hope that this action may be used to interpolate between the regions of small $T$ (vicinity of the tachyon vacuum) and large $T$ where we can expect that tachyon and other open string modes are frozen. Another example of the tachyon effective action is "tachyon DBI" lagrangian

$$S = -\int dtd^p\mathbf{x}V(T)\sqrt{-\det(\eta_{\mu\nu}+\partial_\mu T\partial_\nu T)}\,. \qquad (1.3)$$

As was argued recently in [17] the meaning of (1.3) is not completely clear since there is no reason to expect that higher-dimensional derivative terms omitted in (1.3) should be small on solutions of resulting equations.

We can also try to reconstruct the tachyon effective action at a vicinity of other exact conformal points. One such a conformal point is time-dependent background that represents exact boundary conformal field theory [19]–[22]

$$T = f_0e^{\mu x^0}+\overline{f}_0e^{-\mu x^0}\,, \qquad \mu^2_{\mathrm{bose}} = 1\,, \qquad \mu^2_{super} = \frac{1}{2} \qquad (1.4)$$

or its special case "rolling tachyon" background

$$T = f_0e^{\mu x^0}\,. \qquad (1.5)$$

By demanding that generic first-derivative lagrangian[1]

$$L = -V(T)K(\partial T) \qquad (1.6)$$

should have (1.5)(with $\mu^2 = 1/2$ in the superstring case) as its exact solution fixes its time-dependent part to be [3][2]

$$L = -\frac{1}{1+\frac{T^2}{2}}\sqrt{1+\frac{T^2}{2}-(\partial_0T)^2}\,. \qquad (1.7)$$

If we now assume that (1.7) has direct Lorenz-covariant generalisation we obtain the tachyon effective action

$$S = -\int dtd^p\mathbf{x}\frac{1}{1+\frac{T^2}{2}}\sqrt{1+\frac{T^2}{2}+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}\,. \qquad (1.8)$$

The action (1.8) after field redefinition $\frac{T}{\sqrt{2}} = \sinh\frac{\tilde{T}}{\sqrt{2}}$ becomes

[1]We are using the convention where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,\dots,1), \mu,\nu = 0,\dots,p, a, b = 1,\dots,p, x^0 = t, \mathbf{x} = (x^1,\dots,x^p)$ and the fundamental string tension has been set equal to $(2\pi)^{-1}, \alpha' = 1$.

[2]For similar approach to this problem in the bosonic case, see recent paper [16].



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$$S = -\int dt d^p\mathbf{x} V(\tilde{T})\sqrt{1+\eta^{\mu\nu}\partial_\mu\tilde{T}\partial_\nu\tilde{T}}\,, \qquad V(\tilde{T}) = \cosh\frac{\tilde{T}}{\sqrt{2}}\,. \tag{1.9}$$

which has the form of "DBI" tachyon action (1.3).

As it is clear from previous remarks it is very important to know where in the field theory space different tachyon effective actions correctly describe the tachyon dynamics. For example, even if the rolling tachyon solution $T = e^{\frac{x_0}{\sqrt{2}}}$ is an exact solution of the equation of motion that arises from (1.8) it is not completely clear whether action (1.8) should be used for the description of field theory space around perturbative vacuum $T = 0$. In particular, in [23, 24, 25] analysis of the stability of the the classical time-dependent solution of (1.3) was performed with some interesting results. It was shown that during very short time interval these fluctuations become large and hence could significantly change the classical evolution. One can also interpret the huge grow of fluctuations as a creation of the large number of fluctuation modes in the time-dependent background of the classical solution [26]. Then we can expect that at some time close to the beginning of the tachyon condensation the density of the number of particles created reaches the stringy density and the linearised approximation, where we presume that the fluctuations are small breaks down. In fact, the main goal of this paper is confirmation of this claim.

In order to calculate the number of particle created at any time event $t$ we will formulate the quantum field theory of fluctuation modes in the Schrödinger picture. Since we will be interested in the time intervals very close to the beginning of the rolling tachyon process we will calculate fluctuation effective actions in the regime where the classical tachyon solution is small. We also restrict our analysis to the fluctuation modes that have initial momentum $k^2 > \mu^2$ at far past. Modes with $k^2 < \mu^2$ exponentially grow even at the beginning of the tachyon condensation and significantly change the classical evolution. The study of the dynamics of these modes is very difficult task [27, 28] and it is certainly beyond the scope of this paper. We hope that results obtained in this paper allow us to return to this important problem in near future.

Even if we restrict ourselves to the modes with the positive frequency $\omega_{0\mathbf{k}}^2 = k^2 - \mu^2$ at far past we will get some interesting results. Namely we will find the quantum field theory of fluctuations that is derived from the expansion around the classical solution of the effective action (1.8) has some unfamiliar properties. In particular we will show that for modes with $k^2 > \mu^2$ after short time interval their time-dependent frequencies become imaginary. This fact implies huge growth of the number of particles produced and we can expect the strong backreaction of these modes on the classical solution. This result is consistent with the conclusions presented in [23, 24, 25]. On the other hand in the quantum field theory of fluctuation modes based on the tachyon effective action (1.1) we will not observe such a behaviour. We mean that this is an indication, of course not proof, that the effective action (1.1) may be considered as the appropriate one for description of the tachyon dynamics for small $T$.

This paper is organised as follows. In the next section (2) we will study the quantum field theory of fluctuation modes around the rolling tachyon solution on non-BPS D-brane that is described by effective action (1.8). In section (3) we perform the same analysis for the tachyon effective action (1.1). In conclusion (4) we outline our results and suggest further directions of research. Finally in appendix A we give a brief review of Schrödinger picture description of quantum field theory.

## 2. Non-BPS D-brane effective action

In this section we will study the fluctuation modes around the rolling tachyon solution on non-BPS D-brane in superstring theory. We presume that the effective field theory description of the time-dependent tachyon condensation is described by the action (1.8). Now the equation of motion that arises from (1.8) is

$$-\frac{T}{\left(1+\frac{T^2}{2}\right)^2}\sqrt{\mathbf{B}}+\frac{T}{2(1+\frac{T^2}{2})\sqrt{\mathbf{B}}}-\partial_\mu\left(\frac{\eta^{\mu\nu}\partial_\nu T}{(1+\frac{T^2}{2})\sqrt{\mathbf{B}}}\right)=0\,, \tag{2.1}$$

where

$$\mathbf{B}=1+\frac{T^2}{2}+\eta^{\mu\nu}\partial_\mu T\partial_\nu T\,. \tag{2.2}$$

It is easy to see that the solution of (2.1) that describes the rolling of the tachyon from an unstable minimum $T=0$ at $t=-\infty$ to the stable one $T=\infty$ at $t=\infty$ is

$$T_c(t)=a\sqrt{2}e^{\frac{t}{\sqrt{2}}}\,, \tag{2.3}$$

where $a$ is some constant which can be set to 1 by time translation [12] .

Now we would like to study quantum field theory of fluctuation modes above classical solution (2.3). To do this we write the tachyon field $T$ that appears in (1.8) as

$$T(t,\mathbf{x})=T_c(t)+\phi(t,\mathbf{x})\,, \tag{2.4}$$

where $T_c$ is given in (2.3) and $\phi(t,\mathbf{x})$ is fluctuation field around $T_c$ which is presumed to be small with respect to the classical field $T_c$.

To get an quadratic effective action for fluctuation $\phi$ we insert (2.4) into (1.8) and perform an expansion with respect to $\phi$ up to the second order. Thanks to the time dependence of $T_c$ we generally obtain an action for free massive scalar field $\phi$ where metric and mass term are time-dependent. More precisely, let us write

$$\begin{aligned}
S &= -\int dtd^p\mathbf{x}L(T_c+\phi)\\
&= -\int dtd^p\mathbf{x}L(T_c)-\int dtd^p\mathbf{x}\left[\frac{\delta L(T_c)}{\delta T}\phi+\frac{\delta L(T_c)}{\delta\partial_\mu T}\partial_\mu\phi\right]-\\
&-\frac{1}{2}\int dtd^p\mathbf{x}\left(\frac{\delta^2L(T_c)}{\delta T\delta T}\phi^2+2\frac{\delta^2L(T_c)}{\delta T\delta\partial_\mu T}\phi\partial_\mu\phi+\frac{\delta^2L(T_c)}{\delta\partial_\mu T\delta\partial_\nu T}\partial_\mu\phi\partial_\nu\phi\right)+\cdots\,,
\end{aligned} \tag{2.5}$$

where dots mean terms of higher order in $\phi$. Using equation of motion and integration by parts we can easily show that the expression on the second line in (2.5) is equal to zero.

The quadratic effective action for $\phi$ is then given on the third line in (2.5)

$$S = -\frac{1}{2}\int dtd^p\mathbf{x}\left[G^{\mu\nu}(T_c)\partial_\mu\phi\partial_\nu\phi + M^2(T_c)\phi^2\right],$$
$$G^{\mu\nu}(T_c) = \frac{\delta^2 L(T_c)}{\delta\partial_\mu T\delta\partial_\nu T}, \qquad M^2(T_c) = \frac{\delta^2 L(T_c)}{\delta T\delta T} - \partial_\mu\left[\frac{\delta^2 L(T_c)}{\delta T\delta\partial_\mu T}\right]. \tag{2.6}$$

Let us apply this general description for the tachyon effective action (1.8) and the solution (2.3). After some straightforward calculations we obtain following values of metric components $G^{\mu\nu}(t)$ and mass term $M^2(t)$ that appear in (2.6)

$$\begin{aligned} G^{00} &= -\frac{V^2}{\sqrt{\mathbf{B}}} - \frac{V^2\dot{T}^2}{(\mathbf{B})^{3/2}} = -1\,, \\ G^{ab} &= \frac{V^2\delta^{ab}}{\sqrt{\mathbf{B}}} = \frac{\delta^{ab}}{1+e^{\sqrt{2}t}} \equiv G(t)\delta^{ab}\,, \\ M^2 &= -\frac{1}{2}, \end{aligned} \tag{2.7}$$

where we have used that for (2.3) $\mathbf{B} = 1 + \frac{T^2}{2} - \dot{T}^2 = 1$. As follows from (2.6) and from (2.7) the quadratic action for fluctuation modes has the form of the action for the scalar field in the time-dependent background and consequently we can expect that particles will be produced during the time evolution [29, 30, 31]. In order to study this process it appears to be useful to formulate the quantum field theory of fluctuation modes in the Schrödinger picture description.[3] The main advantage of the Schrödinger picture over other waves to characterise vacuum states is that it describes states explicitly by simple wave functional specified by a single, possibly time-dependent, kernel function satisfying a differential equation with a prescribed boundary conditions. This makes no reference to the assumed spectrum of excited states and so circumvents the difficulties of the conventional canonical description of a vacuum as a no-particle state with respect to the creation and annihilation operators defined by mode decomposition of the field, an approach not well suited to time-dependent problems. In particular, we will see that using explicit time-dependence of the vacuum wave functional one can easily calculate time-dependent vacuum expectation values of the quantum field theory operators. This approach allows us to see at what time the vacuum expectation value of the operator of the number of particles with momentum $\mathbf{k}$ reaches the string density which has strong consequence for further evolution of the system.

Since the brief review of Schrödinger picture description of quantum field theory is given in the appendix now we only mention some basic facts which will be needed in future. Firstly, the wave functional $\Psi$ is function of $\phi(\mathbf{x})$ and explicitly depends on time. Any operator $\hat{O}(\hat{\phi},\hat{\Pi})$ is written in Schrödinger representation as $\hat{O}(\hat{\phi},\hat{\Pi}) = O(-i\frac{\delta}{\delta\phi},\phi)$. Consequently the Schrödinger equation is

$$i\frac{\partial\Psi[\phi,t]}{\partial t} = H\left(-i\frac{\delta}{\delta\phi},\phi\right)\Psi[\phi,t]\,. \tag{2.8}$$

[3] Brief review of Schrödinger picture description of quantum field theory is given in appendix A.

Since in the problems we are considering the spatial sections are flat, it is natural to perform a Fourier transformation as

$$\phi(\mathbf{x}) = \int \frac{d^p\mathbf{k}}{(2\pi)^p} e^{i\mathbf{k}\mathbf{x}} \alpha(\mathbf{k})\,. \tag{2.9}$$

In $k$ space, the hamiltonian operator that corresponds to the lagrangian for fluctuation modes (2.6) and that appears in (2.8) has the form

$$H = \frac{1}{2} \int \frac{d^p\mathbf{k}}{(2\pi)^p} \left[ -\frac{\delta^2}{\delta\alpha(\mathbf{k})\delta\alpha(-\mathbf{k})} + \Omega^2_{\mathbf{k}}(t)\alpha(\mathbf{k})\alpha(-\mathbf{k}) \right], \tag{2.10}$$

where

$$\Omega^2_{\mathbf{k}}(t) = \frac{k^2}{1+e^{\sqrt{2}t}} + m^2\,, \qquad m^2 = -\frac{1}{2}\,. \tag{2.11}$$

We see that for each $k$, the integrand in (2.10) represents a harmonic oscillator with the time-dependent frequency $\Omega^2_{\mathbf{k}}(t)$. Then the vacuum wave functional is

$$\Psi[\alpha, t] = N_0(t) \exp\left[ -\frac{1}{2} \int \frac{d^p\mathbf{k}}{(2\pi)^p} \alpha(-\mathbf{k}) \tilde{G}_{\mathbf{k}}(t) \alpha(\mathbf{k}) \right], \tag{2.12}$$

where the kernel $\tilde{G}_{\mathbf{k}}(t)$ is solution of the equation

$$i\frac{\partial \tilde{G}_{\mathbf{k}}}{\partial t} = \tilde{G}^2_{\mathbf{k}}(t) - \Omega^2_{\mathbf{k}}(t)\,. \tag{2.13}$$

This equation is solved with the ansatz for $\tilde{G}_{\mathbf{k}}$

$$\tilde{G}_{\mathbf{k}}(t) = -i\frac{\dot{\psi}_{\mathbf{k}}(t)}{\psi_{\mathbf{k}}(t)}\,, \tag{2.14}$$

where $\psi_{\mathbf{k}}$ obeys

$$\ddot{\psi}_{\mathbf{k}} + \Omega^2_{\mathbf{k}}(t)\psi_{\mathbf{k}} = 0\,. \tag{2.15}$$

As we show in the appendix the vacuum expectation value of the operator of the number of particles with the momentum $\mathbf{k}$ at time $t$ is equal to

$$\langle N(\mathbf{k}, t)\rangle = (2\pi)^p \delta_{\mathbf{k}}(0) \frac{(\Omega_{\mathbf{k}}(t) - \tilde{G}_{\mathbf{k}}(t))(\Omega_{\mathbf{k}}(t) - \tilde{G}^*_{\mathbf{k}}(t))}{2\Omega_{\mathbf{k}}(t)(\tilde{G}_{\mathbf{k}}(t) + \tilde{G}^*_{\mathbf{k}}(t))}. \tag{2.16}$$

In the previous expression the factor $(2\pi)^p\delta_{\mathbf{k}}(0)$ is the volume of the spatial section hence it is natural to introduce the vacuum expectation value of the density of particles with the momentum $\mathbf{k}$ $\mathcal{N}(\mathbf{k}, t)$ as

$$\langle \mathcal{N}(\mathbf{k}, t)\rangle \equiv \frac{\langle N(\mathbf{k}, t)\rangle}{V_p} = \frac{(\Omega_{\mathbf{k}}(t) - \tilde{G}_{\mathbf{k}}(t))(\Omega_{\mathbf{k}}(t) - \tilde{G}^*_{\mathbf{k}}(t))}{2\Omega_{\mathbf{k}}(t)(\tilde{G}_{\mathbf{k}}(t) + \tilde{G}^*_{\mathbf{k}}(t))}\,. \tag{2.17}$$

Now it is clear that in order to calculate (2.17) at any time $t$ we should find the solution of the equation (2.15) that determines the kernel $\tilde{G}_{\mathbf{k}}$. Since our goal is to determine the vacuum expectation value of the operator of the number of particles in time event near the beginning of the tachyon condensation at $t = -\infty$ we have $e^{\sqrt{2}t} \ll 1$ and hence

$$\Omega^2_{\mathbf{k}}(t) = k^2 - k^2 e^{\sqrt{2}t} + m^2\,. \tag{2.18}$$

Then (2.15) has the form

$$\ddot{\psi}_{\mathbf{k}} - k^2 e^{\sqrt{2}t}\psi_{\mathbf{k}} + (k^2+m^2)\psi_{\mathbf{k}} = 0 \tag{2.19}$$

which can be written as

$$s^2\psi''_{\mathbf{k}} + s\psi'_{\mathbf{k}} - (s^2+n^2)\psi_{\mathbf{k}} = 0 \ , n^2 = -2\omega_{0\mathbf{k}}^2 \, , \tag{2.20}$$

where

$$s = 2\sqrt{\lambda}e^{\frac{t}{\sqrt{2}}} \, , \qquad \lambda = \frac{k^2}{2} \, , \qquad \omega_{0\mathbf{k}}^2 = k^2+m^2 \, . \tag{2.21}$$

The function $\psi_{\mathbf{k}}$ is the solution of the second-order ordinary differential equation and is therefore a linear combination of two independent solutions with arbitrary coefficients. Since the kernel itself is the derivative of the logarithm $\psi_{\mathbf{k}}(t)$ the overall normalisation is unimportant. The kernel therefore depends on one arbitrary parameter which can be found from the boundary conditions. Since the initial configuration corresponds to ordinary unstable D-brane it is natural to choose it that at far past the vacuum wave functional approaches the usual positive frequency Minkowski vacuum

$$\Psi[\phi] \sim \exp\left[-\frac{1}{2}\int \frac{d^p\mathbf{k}}{(2\pi)^p}\alpha(\mathbf{k})\omega_{0\mathbf{k}}\alpha(-\mathbf{k})\right] . \tag{2.22}$$

This boundary condition implies that $\psi_{\mathbf{k}}$ is

$$\psi_{\mathbf{k}}(t) = C_{\mathbf{k}} J_{\sqrt{2}i\omega_{0\mathbf{k}}}\left(2\sqrt{\lambda}ie^{\frac{t}{\sqrt{2}}}\right) . \tag{2.23}$$

To see this note that for $t \to -\infty$ $\psi_{\mathbf{k}}$ has asymptotic form

$$\psi_{\mathbf{k}} \approx e^{i\omega_{0\mathbf{k}}t} \tag{2.24}$$

as follows from the definition of Bessel's functions

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty}\frac{(-1)^k}{k!\Gamma(n+k+1)}\left(\frac{x}{2}\right)^{2k} . \tag{2.25}$$

Then it is easy to see that

$$\tilde{G}_{\mathbf{k}}(-\infty) = \lim_{t\to-\infty}\left(-i\frac{\dot{\psi}_{\mathbf{k}}(t)}{\psi_{\mathbf{k}}(t)}\right) = \omega_{0\mathbf{k}} \, . \tag{2.26}$$

We use these results in the calculation of the vacuum expectation value (2.17) at the time near the beginning of the tachyon condensation. For such a large negative $t$ we can restrict to the terms of the second order in (2.25) and we get

$$\tilde{G}_{\mathbf{k}} = -i\frac{\dot{\psi}_{\mathbf{k}}}{\psi_{\mathbf{k}}} = \omega_{0\mathbf{k}} + \mathcal{D}_{\mathbf{k}}e^{\sqrt{2}t} \, , \qquad \Omega_{\mathbf{k}}^2(t) = \omega_{0\mathbf{k}} - \frac{\lambda e^{\sqrt{2}t}}{\omega_{0\mathbf{k}}} \, , \tag{2.27}$$

where

$$\mathcal{D}_{\mathbf{k}} = \frac{\left(i(\sqrt{2}-1) - \omega_{0\mathbf{k}}\right)\lambda}{1+\sqrt{2}i\omega_{0\mathbf{k}}} \, . \tag{2.28}$$

Then the vacuum expectation value of the operator of the density of particles with momentum $\mathbf{k}$ is equal to

$$\langle \mathcal{N}(\mathbf{k},t)\rangle \approx \frac{1}{2\omega_{0\mathbf{k}}^2}\left(\mathcal{D}_{\mathbf{k}}+\frac{\lambda}{\omega_{0\mathbf{k}}}\right)\left(\mathcal{D}_{\mathbf{k}}^*+\frac{\lambda}{\omega_{0\mathbf{k}}}\right)e^{2\sqrt{2}t}\,. \tag{2.29}$$

However we must stress one important point. As we claimed in the introduction we restrict ourselves to the modes that have positive initial frequency at far past $\omega_{0\mathbf{k}}^2 = k^2 - \frac{1}{2} > 0$ that, according to (2.6) have time-dependent frequencies that for small $t$ are equal to

$$\Omega_{\mathbf{k}}^2 = \omega_{0\mathbf{k}}^2 - k^2 e^{\sqrt{2}t}\,. \tag{2.30}$$

From the upper expression we can deduce that there exists time event $t_*$ when $\Omega_{\mathbf{k}}$ approaches zero

$$\frac{k^2 e^{\sqrt{2}t_*}}{\omega_{0\mathbf{k}}^2} \approx 1 \Rightarrow e^{\sqrt{2}t_*} \approx \left(1-\frac{1}{2k^2}\right). \tag{2.31}$$

We see that for $k^2 \approx 1/2$, $t_* \to -\infty$. The vanishing of $\Omega_{\mathbf{k}}$ at $t_*$ implies that the vacuum expectation value $\langle \mathcal{N}(\mathbf{k},t)\rangle$ diverges at $t_*$. This huge growth of the vacuum expectation value of $\mathcal{N}(\mathbf{k},t)$ for $t \to t_*$ implies the large backreaction on the classical solution and linearised approximation breaks down very quickly after beginning of the tachyon condensation. On the other hand as we will see in section (3) such a behaviour is absent in the case of tachyon effective action (1.1).

In spite of this remark one can use (2.29) for the estimation of the time when the density of particles with $k^2 \gg 1/2$ is of order of the string scale. In this case we have $\omega_{0\mathbf{k}} \approx k, \lambda/\omega_{0\mathbf{k}} \approx k, \mathcal{D}_{\mathbf{k}} \approx ik^2$ and hence

$$\langle \mathcal{N}(\mathbf{k},t)\rangle \approx \frac{1}{k^2}(k+ik^2)(k-ik^2)e^{2\sqrt{2}t} \approx \frac{(k^2+k^4)}{k^2}e^{2\sqrt{2}t} \approx k^2 e^{2\sqrt{2}t} \tag{2.32}$$

so that the time when the $\langle \mathcal{N}(\mathbf{k},t)\rangle$ reaches stringy density is proportional to

$$\langle \mathcal{N}(\mathbf{k},t)\rangle \approx 1 = k^2 e^{2\sqrt{2}t_s} \Rightarrow e^{\sqrt{2}t_s} \approx \frac{1}{k} \ll 1\,, \qquad t_s \approx -\ln k \ll 0 \tag{2.33}$$

that is again close to the beginning of the tachyon condensation. Note that for $t = t_s$ $\Omega_{\mathbf{k}}(t_s)$ is still positive

$$\Omega_{\mathbf{k}}^2(t_s) = \omega_{0\mathbf{k}}^2 - k^2 e^{\sqrt{2}t_s} \approx k(k-1) \gg 1\,. \tag{2.34}$$

To conclude, the fact that the number of particles that are created during very short time evolution of the vacuum state is so large, leads to the breaking of the the linearised approximation very quickly after beginning of the tachyon condensation. Then we can also expect large backreaction of fluctuation modes on the classical solution. This result is in agreement with analysis of fluctuations performed in [23, 24] where several tachyon "DBI" actions were studied.



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## 3. Perturbative tachyon effective action

In this section we will perform the analysis of fluctuations around the rolling tachyon solution of the equation of motion that arises from (1.1)

$$S=-\frac{1}{2}\int dtd^p\mathbf{x}\left[\eta^{\mu\nu}\partial_\mu T\partial_\nu T-\mu^2T^2+gT^4+\cdots\right], \tag{3.1}$$

where $\mu^2_{bose}=1,\mu^2_{super}=1/2$. The linearised equation of motion that follows from (3.1) is

$$\partial_\mu\left(\eta^{\mu\nu}\partial_\nu T\right)+\mu^2T=0 \tag{3.2}$$

that has time-dependent solution

$$T_c=e^{\lambda t}\,, \qquad -\lambda^2+\mu^2=0\,. \tag{3.3}$$

Note that one can trust the linearised approximation for the classical field $T_c$ only if $\mu^2T^2\gg gT^4\Rightarrow\mu^2/g\gg e^{2t}$ and hence we must restrict to the times near the begining of the tachyon condensation when $e^{2t}\ll 1$.

In order to study the dynamics of fluctuations around (3.3) we will proceed as in section (2). From (3.1) we get the metric and mass term in the quadratic action for fluctuation field $\phi$ (2.6)

$$\begin{aligned}G^{\mu\nu}&=\frac{\delta^2L}{\delta\partial_\mu T\partial_\nu T}=-\eta^{\mu\nu}\,,\\ m^2(t)&=\frac{\delta^2L}{\delta T\delta T}=\mu^2-6gT_c^2=\mu^2-6ge^{2\mu t}\,.\end{aligned} \tag{3.4}$$

So that the lagrangian and hamiltonian $H$ are

$$\begin{aligned}L&=-\frac{1}{2}\int dtd^p\mathbf{x}\left[\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-(\mu^2-6ge^{2\mu t})\phi^2\right]\Rightarrow\\ H&=\frac{1}{2}\int d^p\mathbf{x}\left[\dot{\phi}^2+\delta^{ab}\partial_a\phi\partial_b\phi+(6ge^{2\mu t}-\mu^2)\phi^2\right].\end{aligned} \tag{3.5}$$

Then the corresponding operator $\hat{H}$ is

$$\begin{aligned}\hat{H}&=\frac{1}{2}\int\frac{d^p\mathbf{k}}{(2\pi)^p}\left[-\frac{\delta^2}{\delta\alpha(\mathbf{k})\alpha(-\mathbf{k})}+\Omega^2_{\mathbf{k}}(t)\alpha(-\mathbf{k})\alpha(\mathbf{k})\right]\\ \Omega^2_{\mathbf{k}}(t)&=6ge^{2\mu t}+\omega^2_{0\mathbf{k}}\equiv\lambda e^{2\mu t}+\omega^2_{0\mathbf{k}} \qquad ,\omega^2_{0\mathbf{k}}=k^2-\mu^2\,, \qquad \lambda=6g\,.\end{aligned} \tag{3.6}$$

In the following we restrict ourselves to the bosonic case $\mu^2=1$, for supersymetric case $\mu^2=1/2$ the situation will be completely the same. As in previous section we are interested in the form of the kernel $\tilde{G}_{\mathbf{k}}(t)=-i\frac{\dot{f}_{\mathbf{k}}}{f_{\mathbf{k}}}$ where $f_{\mathbf{k}}$ obeys the differential equation

$$\ddot{f}_{\mathbf{k}}+\Omega^2_{\mathbf{k}}f_{\mathbf{k}}=0\,. \tag{3.7}$$

For $\Omega$ given in (3.6) the equation (3.7) has the general solution

$$f_{\mathbf{k}}(t)=Af^{in}_{\mathbf{k}}(t)+Bf^{in*}_{\mathbf{k}}(t)\,, \tag{3.8}$$

where

$$f_{\mathbf{k}}^{in}(t) = J_{-i\omega_{0\mathbf{k}}}(\sqrt{\lambda}e^{t}) \,. \tag{3.9}$$

The condition that the vacuum wave functional approaches for $t \to -\infty$ the usual Minkowski vacuum state functional implies that $A = 0, B = 1$ in (3.8) so that

$$\tilde{G}_{\mathbf{k}}(t) = -i\frac{\dot{f}_{\mathbf{k}}^{in*}}{f_{\mathbf{k}}^{in*}} \,. \tag{3.10}$$

Repeating the same calculations as in section (2) we obtain the vacuum expectation value of the operator of the density of particles with momentum $\mathbf{k}$ at time close to the beginning of the tachyon condensation

$$\langle \mathcal{N}(\mathbf{k},t) \rangle \approx \frac{1}{2}\left(\frac{\lambda}{2\omega_{0\mathbf{k}}^2} + D_{\mathbf{k}}\right)\left(\frac{\lambda}{2\omega_{0\mathbf{k}}^2} + D_{\mathbf{k}}^{*}\right) e^{4t} \,, \tag{3.11}$$

where

$$D = \frac{\lambda}{(1 + i\omega_{0\mathbf{k}})} \,. \tag{3.12}$$

For large $\omega_{0\mathbf{k}} \gg 1$ the upper expression simplifies to

$$\langle \mathcal{N}(\mathbf{k},t) \rangle \approx \frac{\lambda^2}{\omega_{0\mathbf{k}}^4} e^{4t} \,. \tag{3.13}$$

Since the linearised approximation breaks down when $\mu^2 T^2 \approx gT^4 \Rightarrow e^{2t_{lin}} = \frac{6}{\lambda}$ we obtain from (3.13) that for $t = t_{lin}$ the number of particles produced remains still small

$$\langle \mathcal{N}(\mathbf{k},t_{lin}) \rangle \approx \frac{1}{\omega_{0\mathbf{k}}^4} \ll 1 \,. \tag{3.14}$$

We also see that as opposite to the cases of the effective actions discussed in previous sections the time-dependent frequency $\Omega_{\mathbf{k}}^2$ is positive for all times so that the vacuum expectation value of the operator of density of particles do not diverge, or equivalently [23, 24, 25] no new exponentially growing modes are produced during the time evolution. From these facts one can deduce that the effective action (1.1) correctly describes the tachyon dynamics at the beginning of the tachyon condensation.

## 4. Conclusion

In this paper we have calculated the production of fluctuation modes around the rolling tachyon solution on unstable D-brane at times near the beginning of the tachyon condensation. The main purpose of this analysis was to see whether creation of fluctuation modes in the time-dependent background of the classical solution is strong enough to be able to reach the stringy density at finite time from the beginning of the rolling tachyon process. Then we can expect that the linearised approximation breaks down and in order to properly describe the tachyon condensation on D-brane we should study the full non-linear action and also take into account the backreaction of fluctuations on classical solution. Unfortunately these problems are very difficult and their solution is beyond the scope of



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this paper. We also mean that the calculation of the particle production could be useful in the answering the question which form of the tachyon effective action is more suitable for the description of the beginning of the tachyon condensation. In particular, this question seems to be very important in case of effective action (1.8) since its precise meaning is not completely clear. Recently, very nice analysis of the tachyon effective action (1.8) was performed where it was argued that there are problems with the interpretation of (1.8) or more generally with action (1.3). On the other hand it was shown in the recent paper [32] that (1.3) gives description of the moduli space of D-brane compactified on circle of critical radius which is in very good agreement with the exact CFT analysis.

The conclusion given above is directly related to the results we have found in this paper when we have studied the quantum field theory of fluctuation modes around the rolling tachyon solution of the equation of motion that arises from (1.8). In particular, we have shown that for modes, that initially correspond to ordinary fluctuation modes with positive frequencies, their effective time-dependent frequencies vanish at time near the beginning of the tachyon condensation and consequently the vacuum expectation value of the particle density diverges very shortly after beginning of the tachyon condensation. We mean that this result is in agreement with the analysis performed in [23, 24, 25] where it was shown instability of the classical solution of the equation motion arising from (1.3) with respect to the growth of the fluctuation modes. It was argued there that the linear approximation breaks down very quickly after beginning of the rolling tachyon process. On the other hand when we have performed the same analysis in case of the tachyon effective action (1.1) we have found that the particle productions of positive frequency modes is small in the time interval when we can trust the linearised approximation. These results suggest to us that the effective action (1.1) seems to be suitable for the description of the tachyon dynamics at the beginning of the rolling tachyon process.

As we mentioned above to find the regions of validity of various tachyon effective actions is an important problem. Its solving could lead to better understanding of the problem of the tachyon condensation in string theory. It is clear that there are lot of open questions in the effective action description of the tachyon dynamics on D-brane and we also hope that our modest contribution could be helpful in further research in this area.



## A. QFT in Schrödinger picture

In this appendix we briefly review the Schrödinger picture formulation of quantum field theory. For more informations, see [33, 34].

Let us star with the action for free scalar field with time-dependent mass term[4]

$$S=\int dtd^p\mathbf{x}L=-\frac{1}{2}\int dtd^p\mathbf{x}\left(-\partial_t\phi\partial_t\phi+\eta^{ab}\partial_a\phi\partial_b\phi+m^2(t)\phi^2\right). \tag{A.1}$$

The canonical momentum conjugate to $\phi$ is

$$\Pi(t,\mathbf{x})=\frac{\delta L}{\delta\dot{\phi}}=\dot{\phi}\,, \tag{A.2}$$

[4]Our convention is $\eta_{\mu\nu}=\mathrm{diag}(-1,\dots,1),\mu,\nu=0,\dots,p,x^0=t,\mathbf{x}=(x^1,\dots,x^p)$.

and the hamiltonian

$$H = \int d^p\mathbf{x}\left(\Pi\dot{\phi} - L\right) = \frac{1}{2}\int d^p\mathbf{x}\left(\Pi^2 + \eta^{ab}\partial_a\phi\partial_b\phi + m^2(t)\phi^2\right). \tag{A.3}$$

The system can be canonically quantised by treating the fields as operators and imposing appropriate commutation relations. This involves choice of a foliation of a space-time into a succession of spacelike hypersurfaces. We choose these to be the hypersurface of fixed $t$ and impose equal time commutation relations

$$[\hat{\phi}(\mathbf{x},t), \hat{\Pi}(\mathbf{y},t)] = i\delta(\mathbf{x}-\mathbf{y}), \qquad [\hat{\phi}(\mathbf{x},t), \hat{\phi}(\mathbf{y},t)] = [\hat{\Pi}(\mathbf{x},t), \hat{\Pi}(\mathbf{y},t)] = 0. \tag{A.4}$$

In the Schrödinger picture we take the basis vector of the state vector space to be the eigenstate of the field operator $\hat{\phi}(t,\mathbf{x})$ on a fixed $t$ hypersurface, with eigenvalues $\phi(\mathbf{x})$

$$\hat{\phi}(t,\mathbf{x})\left|\phi(\mathbf{x}),t\right\rangle = \phi(\mathbf{x})\left|\phi(\mathbf{x}),t\right\rangle. \tag{A.5}$$

Notice that the set of field eigenvalues $\phi(\mathbf{x})$ is independent of the value of $t$ labelling the hypersurface. In this picture, the quantum states are explicit functions of time and are represented by wave functionals $\Psi[\phi(\mathbf{x}),t]$. Operators $\hat{\mathcal{O}}(\hat{\phi},\hat{\Pi})$ acting on these states may be represented by

$$\hat{\mathcal{O}}(\hat{\Pi}(x),\hat{\phi}(x)) = \mathcal{O}\left(-i\frac{\delta}{\delta\phi(\mathbf{x})}, \phi(\mathbf{x})\right). \tag{A.6}$$

The Schrödinger equation which governs the evolution of the wave functional between the spacelike hypersurfaces, is

$$\begin{aligned} i\frac{\partial\Psi[\phi,t]}{\partial t} &= H\left(-i\frac{\delta}{\delta\phi(\mathbf{x})}, \phi(\mathbf{x})\right)\Psi[\phi,t] \\ &= \frac{1}{2}\int d^p\mathbf{x}\left[-\frac{\delta^2}{\delta\phi^2} + \eta^{ab}\partial_a\phi\partial_b\phi + m^2(t)\phi^2\right]\Psi[\phi,t]. \end{aligned} \tag{A.7}$$

To solve this equation, we make the ansatz, that up to time-dependent phase, the vacuum functional is simple gaussian. We therefore write

$$\Psi_0[\phi,t] = N_0(t)\exp\left\{-\frac{1}{2}\int d^p\mathbf{x}d^p\mathbf{y}\phi(\mathbf{x})G(\mathbf{x},\mathbf{y},t)\phi(\mathbf{y})\right\} = N_0(t)\psi_0(\phi,t), \tag{A.8}$$

where $N_0(t)$ and $G(\mathbf{x},\mathbf{y},t)$ obey

$$\begin{aligned} i\frac{\partial N_0(t)}{\partial t} &= \frac{N_0(t)}{2}\int d^p\mathbf{z}G(\mathbf{z},\mathbf{z},t), \\ i\frac{\partial G(\mathbf{x},\mathbf{y},t)}{\partial t} &= \int d^p\mathbf{z}G(\mathbf{z},\mathbf{x},t)G(\mathbf{y},\mathbf{z},t) - \left(\eta^{ab}\partial_a\partial_b + m^2(t)\right)\delta(\mathbf{x},\mathbf{y}). \end{aligned} \tag{A.9}$$

For situations, where the spatial sections are flat, it is natural to perform a Fourier transformation on the space dependence of field configuration

$$\phi(\mathbf{x}) = \int\frac{d^p\mathbf{k}}{(2\pi)^p}e^{i\mathbf{k}\mathbf{x}}\alpha(\mathbf{k}) \tag{A.10}$$



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Similarly we can define $\delta/\delta\alpha(\mathbf{k})$ as

$$\frac{\delta}{\delta\phi(\mathbf{x})} = \int \frac{d^p\mathbf{k}}{(2\pi)^p} e^{i\mathbf{k}\mathbf{x}} \frac{\delta}{\delta\alpha(\mathbf{k})} . \tag{A.11}$$

Reality of $\phi$ implies $\alpha^*(\mathbf{k}) = \alpha(-\mathbf{k})$ and since

$$\frac{\delta\phi(\mathbf{x})}{\delta\phi(\mathbf{x}')} = \delta(\mathbf{x} - \mathbf{x}') \tag{A.12}$$

we also have

$$\frac{\delta\alpha(\mathbf{k})}{\delta\alpha(\mathbf{k}')} = (2\pi)^p \delta(\mathbf{k} + \mathbf{k}') . \tag{A.13}$$

In $k$ space, the hamiltonian is

$$H = \frac{1}{2} \int \frac{d^p\mathbf{k}}{(2\pi)^p} \left[ -\frac{\delta^2}{\delta\alpha(\mathbf{k})\delta\alpha(-\mathbf{k})} + \Omega_{\mathbf{k}}^2(t)\alpha(\mathbf{k})\alpha(-\mathbf{k}) \right] , \tag{A.14}$$

where

$$\Omega_{\mathbf{k}}^2(t) = m^2(t) + \omega_{0\mathbf{k}}^2 , \qquad \omega_{0\mathbf{k}}^2 = k^2 + m^2 . \tag{A.15}$$

The Fourier transform of the kernel $G(\mathbf{x}, by, t)$ is

$$G(\mathbf{x}, \mathbf{y}, t) = \int \frac{d^p\mathbf{k}}{(2\pi)^p} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \tilde{G}(\mathbf{k}, t) \tag{A.16}$$

and hence the kernel equation (A.9) reduces to

$$i\frac{\partial \tilde{G}(\mathbf{k}, t)}{\partial t} = \tilde{G}^2(\mathbf{k}, t) - \Omega_{\mathbf{k}}^2(t) , \qquad \Omega_{\mathbf{k}}^2(t) = k^2 + m^2(t) \tag{A.17}$$

which can be solved with the ansatz

$$\tilde{G}(\mathbf{k}, t) = -i \frac{\dot{\psi}_{\mathbf{k}}(t)}{\psi_{\mathbf{k}}(t)} , \tag{A.18}$$

where $\psi_{\mathbf{k}}(t)$ obeys

$$\ddot{\psi}_{\mathbf{k}} + \Omega_{\mathbf{k}}^2(t)\psi_{\mathbf{k}} = 0 \tag{A.19}$$

Note that (A.19) is differential equation of second order so that its general solution is given as linear combination of two solutions and hence is parametrised by two independent constants. On the other hand $\psi$ appears in $\tilde{G}$ only in combination $\frac{\dot{\psi}_{\mathbf{k}}}{\psi_{\mathbf{k}}}$ and hence overall normalisation constant is unimportant. As a result the vacuum functional depends on one free parameter. The value of this parameter can be fixed from the initial conditions that vacuum wave functional should obey.

In order to define concept of operator of particles in the quantum field theory with time-dependent mass we will proceed as follows. Let us define following set of operators

$$\begin{aligned} \mathcal{A}(\mathbf{k}, t) &= \frac{1}{\sqrt{2\Omega_{\mathbf{k}}}} \left[ \frac{\delta}{\delta\alpha(\mathbf{k})} + \Omega_{\mathbf{k}}(t)\alpha(\mathbf{k}) \right] \\ \mathcal{A}^\dagger(\mathbf{k}, t) &= \frac{1}{\sqrt{2\Omega_{\mathbf{k}}}} \left[ -\frac{\delta}{\delta\alpha(-\mathbf{k})} + \Omega_{\mathbf{k}}(t)\alpha(-\mathbf{k}) \right] \\ \mathcal{A}^\dagger(\mathbf{k}, t)\mathcal{A}(\mathbf{k}, t) &= \frac{1}{2\Omega_{\mathbf{k}}} \left[ -\frac{\delta^2}{\delta\alpha(-\mathbf{k})\alpha(\mathbf{k})} + \Omega_{\mathbf{k}}^2(t)\alpha(\mathbf{k})\alpha(-\mathbf{k}) - \Omega_{\mathbf{k}}(t)(2\pi)^p\delta_{\mathbf{k}}(0) \right] . \end{aligned} \tag{A.20}$$

With the help of (A.20) we can rewrite the hamiltonian as

$$H(t) = \frac{1}{2}\int \frac{d^p\mathbf{k}}{(2\pi)^p}\Omega_{\mathbf{k}}(t)\left[\mathcal{A}^\dagger(\mathbf{k},t)\mathcal{A}(\mathbf{k},t) + V_p\right], \qquad V_p = (2\pi)^p\delta_{\mathbf{k}}(0) \tag{A.21}$$

that has the form of sum of an infinite number of hamiltonians of harmonic oscillators with the time-dependent frequencies $\Omega_{\mathbf{k}}(t)$ where one can interpret $\mathcal{A}(\mathbf{k},t)(\mathcal{A}^\dagger(\mathbf{k},t))$ as operators that annihilate(create) particle with momentum $\mathbf{k}$ at time $t$. According to this interpretation the operator of the number of particles with momentum $\mathbf{k}$ at time $t$ is

$$N(\mathbf{k},t) = \mathcal{A}^\dagger(\mathbf{k},t)\mathcal{A}(\mathbf{k},t)\,. \tag{A.22}$$

We are mainly interested in the vacuum expectation value of this operator $\langle N_{\mathbf{k}}(t)\rangle$. It can be shown [33, 34][5] that its vacuum expectation value is equal to

$$\langle N(\mathbf{k},t)\rangle = (2\pi)^p\delta_{\mathbf{k}}(0)\frac{\left(\Omega_{\mathbf{k}}(t) - \tilde{G}(\mathbf{k},t)\right)\left(\Omega_{\mathbf{k}}(t) - \tilde{G}^*(\mathbf{k},t)\right)}{2\Omega_{\mathbf{k}}(t)(\tilde{G}(\mathbf{k},t) + \tilde{G}^*(\mathbf{k},t))}\,. \tag{A.23}$$

Since the factor $(2\pi)^p\delta_{\mathbf{k}}(0) = V_p$ is volume of the spatial section of Dp-brane it is natural to define the vacuum expectation value of the operator of density of particles with momentum $\mathbf{k}$ at time $t$ $\mathcal{N}(\mathbf{k},t)$ as

$$\langle\mathcal{N}(\mathbf{k},t)\rangle \equiv \frac{\langle N(\mathbf{k},t)\rangle}{V_p} = \frac{\left(\Omega_{\mathbf{k}}(t) - \tilde{G}(\mathbf{k},t)\right)\left(\Omega_{\mathbf{k}}(t) - \tilde{G}^*(\mathbf{k},t)\right)}{2\Omega_{\mathbf{k}}(t)(\tilde{G}(\mathbf{k},t) + \tilde{G}^*(\mathbf{k},t))}\,. \tag{A.24}$$

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